

Var 01

SUBIECTUL I		(30 de puncte)
1.	$x_V = 3, f(x_V) = 1$ $f(2) = f(4) = 0$ Finalizare $f([2;4]) = [0;1]$	2p 2p 1p
2.	$ z = \left \left(\frac{2-i}{1+2i} \right)^{2013} \right = \left \left(\frac{2-i}{1+2i} \right) \right ^{2013} =$ $\left \frac{2-i}{1+2i} \right = 1 \Rightarrow$ $ z = 1$	2p 2p 1p
3.	$T_{k+1} = C_{13}^k (\sqrt[3]{x})^{13-k} \left(\frac{1}{\sqrt[3]{x^2}} \right)^k \Rightarrow$ $\Rightarrow \frac{13-k}{2} - \frac{2k}{3} = 3 \Rightarrow$ $k = 3$	1p 2p 2p
4.	$A_6^3 - A_5^2 =$ $= 6 \cdot 5 \cdot 4 - 5 \cdot 4 = 100$	3p 2p
5.	$m_{AB} = \frac{y_A - y_B}{x_A - x_B} = \frac{1}{3} \Rightarrow$ $m_d = -\frac{1}{m_{AB}} = -3$ Ecuația lui $d: y - y_C = m_d(x - x_C) \Rightarrow y + 4 = -3x$.	1p 2p 2p
6.	$\cos 165^\circ + \sin 75^\circ = \cos(180^\circ - 15^\circ) + \sin 75^\circ =$ $= -\cos 15^\circ + \sin 75^\circ =$ $= -\sin 75^\circ + \sin 75^\circ = 0$	2p 2p 1p

SUBIECTUL II		(30 de puncte)
1.a.	$a = 0 \Rightarrow I_3 \in G$ $a_{22} = 1 \neq 0$ oricare ar fi $a > -1 \Rightarrow O_3 \notin G$	2p 3p
b.	$\begin{pmatrix} 1+a & 0 & 0 \\ 0 & 1 & 0 \\ ka & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1+b & 0 & 0 \\ 0 & 1 & 0 \\ kb & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+a+b+ab & 0 & 0 \\ 0 & 1 & 0 \\ ka+kb+kab & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 1+b & 0 & 0 \\ 0 & 1 & 0 \\ kb & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1+a & 0 & 0 \\ 0 & 1 & 0 \\ ka & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+b+a+ba & 0 & 0 \\ 0 & 1 & 0 \\ kb+ka+kba & 0 & 1 \end{pmatrix}$	2p 2p 1p
	Finalizare	
c.	$P(n): A^n(a) = A \left((a+1)^n - 1 \right)$, oricare ar fi n natural nenul. Verificare pentru $n=1$	2p

	$A^{n+1}(a) = A^n(a) \cdot A(a)$ Finalizare	3p
2.a.	$x * y = xy - \frac{1}{2}(x+y) + \frac{3}{4} = x\left(y - \frac{1}{2}\right) - \frac{1}{2}\left(y - \frac{1}{2}\right) + \frac{1}{2} =$ $= \left(x - \frac{1}{2}\right) \cdot \left(y - \frac{1}{2}\right) + \frac{1}{2}$	2p 3p
b.	<p>Exista $e \in G$, astfel incat $x * e = e * x = x$ pentru orice $x \in G$</p> $x * e = x \Rightarrow \left(x - \frac{1}{2}\right) \cdot \left(e - \frac{1}{2}\right) + \frac{1}{2} = x \Rightarrow$ $\Rightarrow e = \frac{3}{2}.$	1p 2p 2p
c.	$x * x * x * x = \frac{9}{16} \Leftrightarrow \left(x - \frac{1}{2}\right)^4 + \frac{1}{2} = \frac{9}{16}$ <p>Finalizare: $x = 1$</p>	3p 2p

SUBIECTUL III

(30 de puncte)

1.a.	$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{\ln x}{x-1} \stackrel{x-1=y}{=} \lim_{y \rightarrow 0} \frac{\ln(y+1)}{y} = 1$ f continuă în 1 f continuă pe $(0,1) \cup (1,\infty)$ Finalizare	3p 1p 1p
b.	$\lim_{x \rightarrow 1} \frac{f(x)-1}{x-1} = \lim_{x \rightarrow 1} \frac{\ln x - x + 1}{x-1} \stackrel{l'H}{=}$ $= \lim_{x \rightarrow 1} \frac{1-x}{2x(x-1)} = -\frac{1}{2}$	2p 3p
c.	$f'(x) = \begin{cases} \frac{x-1-x \ln x}{x(x-1)^2}, & x \in (0,1) \cup (1,\infty) \\ -\frac{1}{2}, & x=1 \end{cases}$ $h(x) = x-1-x \ln x < 0$ <p>Finalizare</p>	2p 2p 1p
2.a.	$\int 2x \cdot f_n^2(x) dx = \int \frac{2x}{2x+5n} dx = \int 1 - \frac{5n}{2x+5n} dx =$ $= x - \frac{5n}{2} \ln(2x+5n) + C$	2p 3p
b.	$\int \frac{1}{\sqrt{2x+5}} dx = \int (\sqrt{2x+5})' dx =$ $= \sqrt{2x+5} \Big _0^1 = \sqrt{7} - \sqrt{5}$	3p 2p

c.	$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} (f_n(1) + f_n(2) + \dots + f_n(n)) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{1}{\sqrt{2\frac{k}{n} + 5}} =$ $= \int_0^1 \frac{1}{\sqrt{2x+5}} dx = \sqrt{7} - \sqrt{5}$	3p
		2p